Because the Earth "in the beginning of time" was (and essentially still is) a rotating semi-rigid viscous body it has taken a slightly oblate shape such that the gravitational force is somewhat tilted towards North in the Northern Hemisphere and towards South in the Southern Hemisphere. This prevents a "loose" mass element at rest on the Earth surface from drifting away towards the equator! This is illustrated in figure 1. But atmosphere that is moving relative to the Earth surface (wind!) will not be completly in this equlibrium and there will be some "oscillations" around this equlibrium point. This the "Coriolis Phenomenon" that causes the air to move in whirls rotating in the "clock-sense" in the Northern Hemisphere and in the "counter-clock-sense" in the Northern Hemisphere.

The "centrifugal force" caused by the Earth rotation is

$\varphi^2 r$

where

- φ is the rotation rate of the Earth (one complete revolution in one sidereal day which is 23 h 56 m 4.1 s)
- r is the distance to the Earth axis

The tangential component of the gravitational force required to prevent the mass element to slip away towards the equator is

$\sin \delta r \varphi^2$

where

- δ is the latitude

The mathematical model for the "Coriolis Effect" is a mass point sliding on the surface of the Earth affected by the conservative tangential gravitational force directed towards the closest pole (the North Pole in the Northern hemisphere, the South Pole in the Southern Hemisphere). The equation of motion then takes the form

$$\frac{d^2\mathbf{R}}{d^2t} = \sin \delta \varphi^2 r \mathbf{N} \tag{1}$$

where **N** is "Direction North". Note that this expression applies both in the Northern and the Southern hemisphere as the sign of $\sin\delta$ in equation (1) assures the correct direction of the gravitational force!

Introducing a rectangular coordinate system with origin in Earth center and the Z-axis aligned with the rotation axis one has that

$$\begin{aligned} \mathbf{N} &= (-\cos\alpha\sin\delta, -\sin\alpha\sin\delta, \cos\delta) \\ \mathbf{E} &= (-\sin\alpha, \cos\alpha\ , 0) \end{aligned} \tag{2}$$

where

α is the longitude of the point

δ is the latitude of the point

N is the unit vector in the tangential plane towards north

E is the unit vector in the tangential plane towards east

In this case it is advantageous to use a rotating non-inertial coordinate system. In a rotating coordinate system the equation of motion takes the form

$$\frac{d^2\mathbf{R}}{d^2t} = -2\omega\mathbf{Z} \times \frac{d\mathbf{R}}{dt} + \omega^2\mathbf{R}_p + \mathbf{F} \tag{3}$$

where

R is the position vector

Z is the unit vector defining the rotational axis of the rotating coordinate system

ω is the rotation rate of the rotating coordinate system

$\mathbf{R}_p$ is the projektion of the position vector on the plane orthogonal to **Z**

F is the physical force acting on the mass point

Only the components in the tangential plane matter for the motion of the mass point sliding over the surface. In what follows is should be understood that these vectors always refer to the PROJECTION of these vectors to the tangent plane as the components orthogonal to the surface are counteracted by the "constraint forces" fixing the mass point to the surface of the ellipsoid.

From figure (1) is seen that the horisontal component of the sum

$\omega^2 \mathbf{R}_p + \mathbf{F}$

is zero everywhere on the Earth surface.

Introducing a rotating rectangular coordinat system with origin at the center of the Earth and with the Z axis aligned with the rotational axis of the Earth the unit vector **Z** in (3) is (0,0,1) and the expression (3) therefore takes the form

$$\begin{aligned} \frac{d^2\mathbf{R}}{d^2t} &= 2\omega\mathbf{C} \\ \mathbf{C} &= (-v_y, v_x, 0) \end{aligned} \tag{4}$$

where

v_x is the x-component of the velocity in the rectangular coordinate system

v_y is the x-component of the velocity in the rectangular coordinate system

From (2) follows that the component of **C** in tangential Northern direction is

$C_n = -\cos\alpha \sin\delta v_y + \sin\alpha \sin\delta v_x$

and in tangential Eastern direction

$C_e = -\sin\alpha v_y - \cos\alpha v_x$

The velocity vector of the sliding mass point can be written

$$\mathbf{v} = v_n\mathbf{N} + v_e\mathbf{E} \tag{5}$$

From (2) and (5) follows that

$$v_x = -\cos\alpha \sin\delta v_n - \sin\alpha v_e \tag{6}$$

$$v_y = -\sin\alpha \sin\delta v_n + \cos\alpha v_e \tag{7}$$

Substituting these expressions for v_x and for v_y in (4) one gets after some simplification

$$\frac{d^2\mathbf{R}}{d^2t} = 2\omega \sin\delta(-v_e\mathbf{N} + v_n\mathbf{E}) \tag{8}$$

As the motion of the mass point is

$$\mathbf{v} = v_n\mathbf{N} + v_e\mathbf{E} \tag{9}$$

this means that the projection of the "Coriolis Term" to the tangential plane makes an angle of 90 deg to the direction of motion of the mass point and the magnitude of the projection is $C = 2\omega\mathbf{V}\sin\delta$ where $\mathbf{V}$ is the velocity of the mass point. On the Northern Hemisphere $\sin\delta$ is positive and the Coriolis Term causes a "turn to the right" while in the Southern Hemisphere it instead causes a "turn to the left". The velocity (relative to the rotating frame!) stays constant. If it had not been for the latitude dependent factor $\sin\delta$ this would have been the differential equation for a mass point on a sphere with radius R circulating on a "small circle" with the "half opening angle" of β where

$\tan\beta = \frac{V}{2\omega R\sin\delta_0}$

and $\sin\delta_0$ is sinus for some "close latitude"! It should also be be pointed out that ωR is the velocity of a point fixed to the equator. For the Earth this velocity is 465 m/s. As an example, in the area around 40 deg North atmosphere moving with 30 m/s relative the ground would tend to circulate on a "small circle" with an opening angle of 2.87 deg what corresponds to a radius of 320 km.

It should be pointed out that for the "real motion" of the mass point relative to inertial space it is just a mass particle sliding on the surface of the Earth circulating a centre of gravitational attraction! The velocity relative inertial space certainly is not constant! If for example the mass point would be released without having any (inertial) east/west velocity at all the mass point would be accelerate straight towards the pole with the sum of the potential and the kinetic energies remaining constant. Relative to inertial space the mass point is simply circulating around the pole being the center of gravitational attraction following a somewhat "windling" path!